OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4771

Decision Mathematics 1

Monday **20 JUNE 2005** Morning 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- There is an **insert** for use in Questions **1, 2** and **4**.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 72.

This question paper consists of 8 printed pages and an insert.

Section A (24 marks)

1 Answer this question on the insert provided.

The nodes in the unfinished graph in Fig. 1 represent six components, A, B, C, D, E, F and the mains. The components are to be joined by electrical cables to the mains. Each component can be joined directly to the mains, or can be joined via other components.

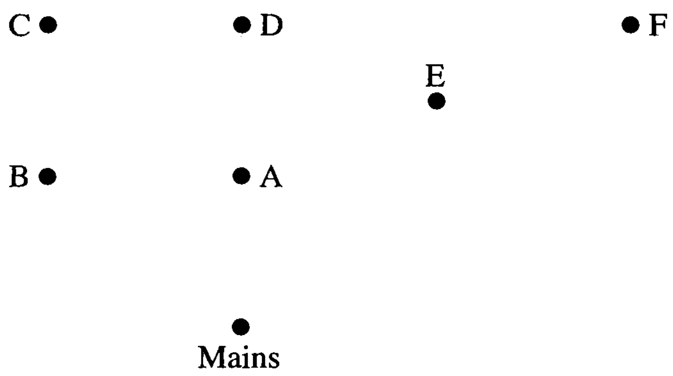

Fig. 1

The total number of connections that the electrician has to make is the sum of the orders of the nodes in the finished graph.

(i) Draw arcs representing suitable cables so that the electrician has to make as few connections as possible. Give this number. [3]

The electrician has a junction box. This can be represented by an eighth node on the graph.

(ii) What is the minimum number of connections which the electrician will have to make if he uses the junction box? [1]

(iii) The electrician has to make more connections if he uses his junction box. Why might he choose to use it anyway? [1]

The electrician decides not to use the junction box. He measures each of the distances between pairs of nodes, and records them in a complete graph. He then constructs a minimum connector for his graph to find which nodes to connect.

(iv) Will this result in the minimum number of connections? Justify your answer. [3]

2 **Answer this question on the insert provided.**

A maze is constructed by building east/west and north/south walls so that there is a route from the entrance to the exit. The maze is shown in Fig. 2.1.

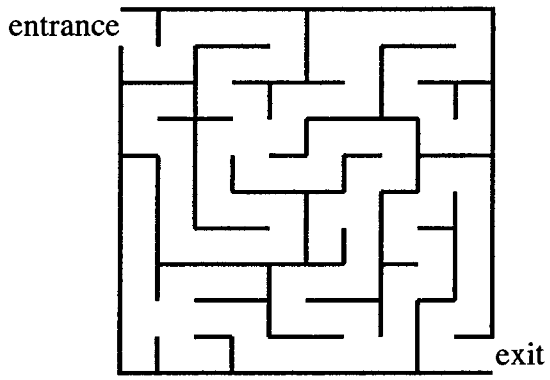

Fig. 2.1

On entering the maze Janet says "I'm always going to keep a hand in contact with the wall on the right." John says "I'm always going to keep a hand in contact with the wall on the left."

(i) On the insert for this question show Janet's route through the maze.

On the insert show John's route. [3]

(ii) Will these strategies always find a way through such mazes? Justify your answer. [2]

In some mazes the objective is to get to a marked point before exiting. An example is shown in Fig. 2.2, where **X** is the marked point.

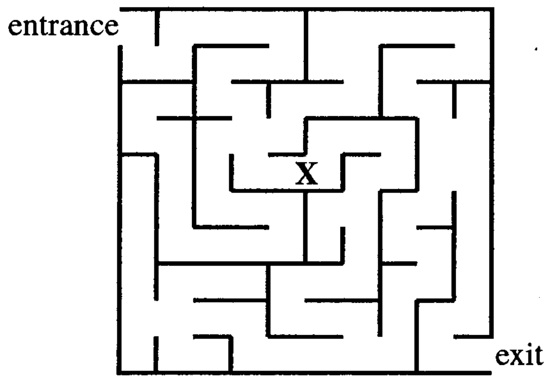

Fig. 2.2

In the maze shown in Fig. 2.2 Janet's algorithm takes her to **X**. John's algorithm does not take him to **X**. John modifies his algorithm by saying that he will turn his back on the exit if he arrives there without visiting **X**. He will then move onwards, continuing to keep a hand in contact with the wall on the left.

(iii) Will this modified algorithm take John to the point **X** in the maze in Fig. 2.2? [1]

(iv) Will this modified algorithm take John to **any** marked point in the maze in Fig. 2.2? Justify your answer. [2]

3 Table 3 gives the durations and immediate predecessors for the five activities of a project.

Activity	Duration (hours)	Immediate predecessor(s)
A	3	–
B	2	–
C	5	–
D	2	A
E	1	A, B

Table 3

(i) Draw an activity-on-arc network to represent the precedences. [2]

(ii) Find the early and late event times for the vertices of your network, and list the critical activities. [3]

(iii) Give the total and independent float for each activity which is not critical. [3]

Section B (48 marks)

4 Answer parts (i) and (ii) on the insert provided.

Fig. 4 shows a network of roads giving direct connections between a city, C, and 7 towns labelled P to V. The weights on the arcs are distances in kilometres. The local coastline is also shown.

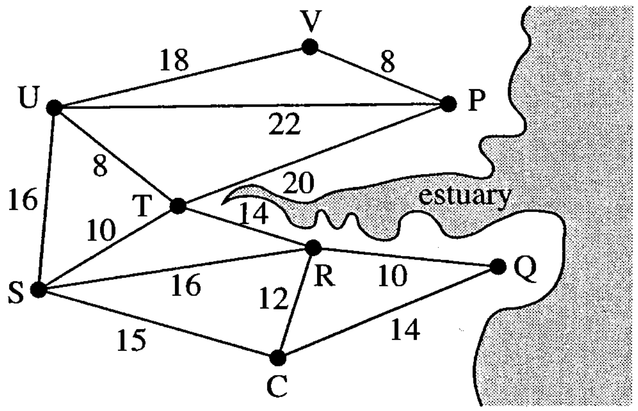

Fig. 4

(i) Use Dijkstra's algorithm on the insert to find the shortest distances from each of the towns to the city, C. List those distances and give the shortest route from P to C and from V to C. [8]

(ii) Use Kruskal's algorithm to find a minimum connector for the network. List the order in which you include arcs and give the length of your connector. [4]

A bridge is built giving a direct road between P and Q of length 12 km.

(iii) What effect does the bridge have on the shortest distances from the towns to the city? (You do not need to use an algorithm to answer this part of the question.) [1]

(iv) What effect does the bridge have on the minimum connector for the network? (You do not need to use an algorithm to answer this part of the question.) [1]

(v) Before the bridge was built it was possible to travel from P to C using every road once and only once. With the bridge in place, it is possible to travel from a different town to C using every road once and only once.

Give this town and justify your answer. [2]

[Turn over

5 A computer store has a stock of 10 laptops to lend to customers while their machines are being repaired. On any particular day the number of laptop loans requested follows the distribution given in Table 5.1.

Number requested	0	1	2	3	4
Probability	0.20	0.30	0.20	0.15	0.15

Table 5.1

(i) Give an efficient rule for using two-digit random numbers to simulate the daily number of requests for laptop loans. [2]

(ii) Use two-digit random numbers from the list below to simulate the number of loans requested on each of ten successive days.

Random numbers: 23, 02, 57, 80, 31, 72, 92, 78, 04, 07 [2]

The number of laptops returned from loan each day is modelled by the distribution given in Table 5.2, independently of the number on loan (which is always at least 5).

Number returned	0	1	2	3
Probability	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{3}$

Table 5.2

(iii) Give an efficient rule for using two-digit random numbers to simulate the daily number of laptop returns. [2]

(iv) Use two-digit random numbers from the list below to simulate the number of returns on each of ten successive days.

Random numbers: 32, 98, 01, 32, 14, 21, 32, 71, 82, 54, 47 [2]

At the end of day 0 there are 7 laptops out on loan and 3 in stock. Each day returns are made in the morning and loans go out in the afternoon. If there is no laptop available the customer is disappointed and never gets a loaned laptop.

(v) Use your simulated numbers of requests and returns to simulate what happens over the next 10 days. For each day record the day number, the number of laptops in stock at the end of the day, and the number of customers that have to be disappointed. [3]

To try to avoid disappointing customers, if the number of laptops in stock at the end of a day is 2 or fewer, the store sends out e-mails to customers with loaned laptops asking for early return if possible. This changes the return distribution for the next day to that given in Table 5.3.

Number returned	0	1	2	3	4
Probability	0.1	0.1	0.4	0.2	0.2

Table 5.3

(vi) Simulate the 10 days again, but using this new policy.

Use the requests you produced in part **(ii)**. Use the random numbers given in part **(iv)** to simulate returns, but use either the distribution given in Table 5.2 or that given in Table 5.3, depending on the number of laptops in stock at the end of the previous day.

Is the new policy better? [5]

[Question 6 is printed overleaf.]

6 A company manufactures two types of potting compost, Flowerbase and Growmuch. The weekly amounts produced of each are constrained by the supplies of fibre and of nutrient mix. Each litre of Flowerbase requires 0.75 litres of fibre and 1 kg of nutrient mix. Each litre of Growmuch requires 0.5 litres of fibre and 2 kg of nutrient mix. There are 12 000 litres of fibre supplied each week, and 25 000 kg of nutrient mix.

The profit on Flowerbase is 9p per litre. The profit on Growmuch is 20p per litre.

(i) Formulate an LP to maximise the weekly profit subject to the constraints on fibre and nutrient mix. [6]

(ii) Solve your LP using a graphical approach. [6]

(iii) Consider each of the following separate circumstances.

(A) There is a reduction in the weekly supply of fibre from 12000 litres to 10000 litres. What effect does this have on profit? [1]

(B) The price of fibre is increased. Will this affect the optimal production plan? Justify your answer. [2]

(C) The supply of nutrient mix is increased to 30000 kg per week. What is the new profit? [1]

Candidate Name	Centre Number	Candidate Number

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

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MEI STRUCTURED MATHEMATICS 4771

Decision Mathematics 1

INSERT

Monday **20 JUNE 2005** Morning 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

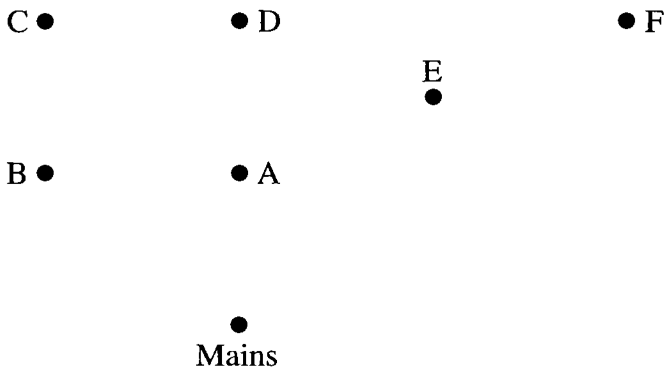
- This insert should be used in Questions **1**, **2** and **4**.
- Write your name, centre number and candidate number in the spaces provided at the top of this page and attach it to your answer booklet.

This insert consists of 4 printed pages.

HN/3

[Turn over

1 **(i)**



Minimum number of connections: __________

(ii) Minimum number of connections using junction box: __________

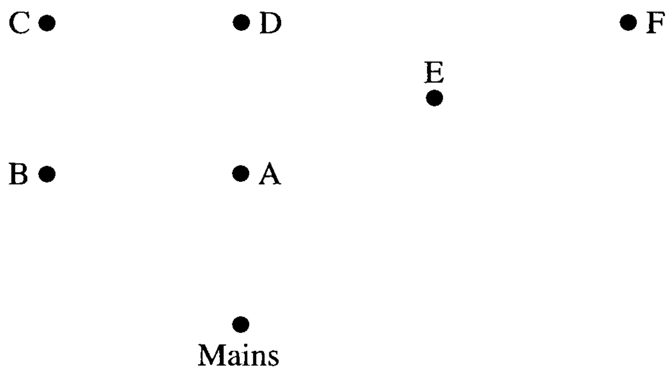
(iii) ____

____

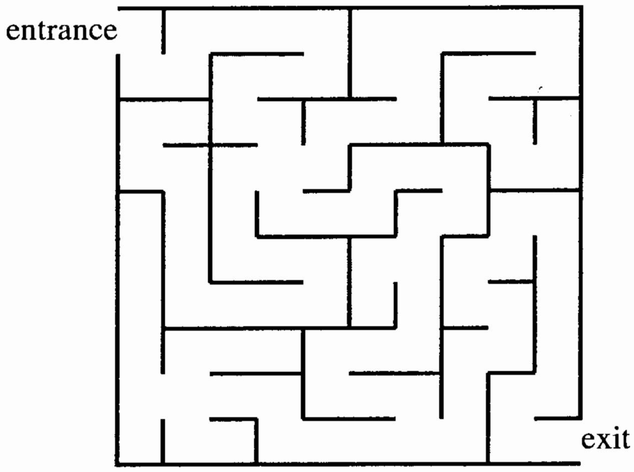
(iv) ____

____

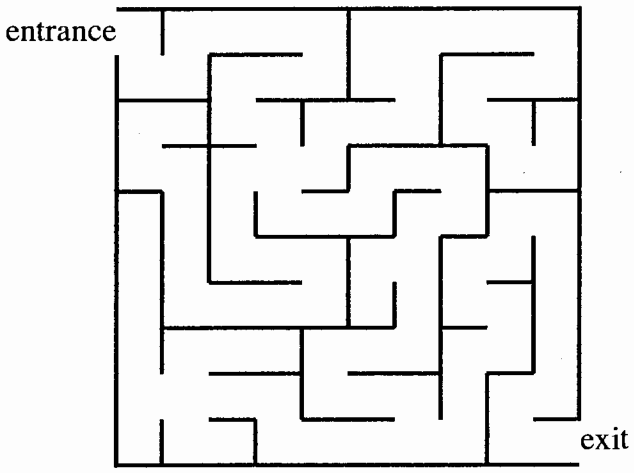
Spare copy of diagram



2 **(i)** Janet's route



John's route

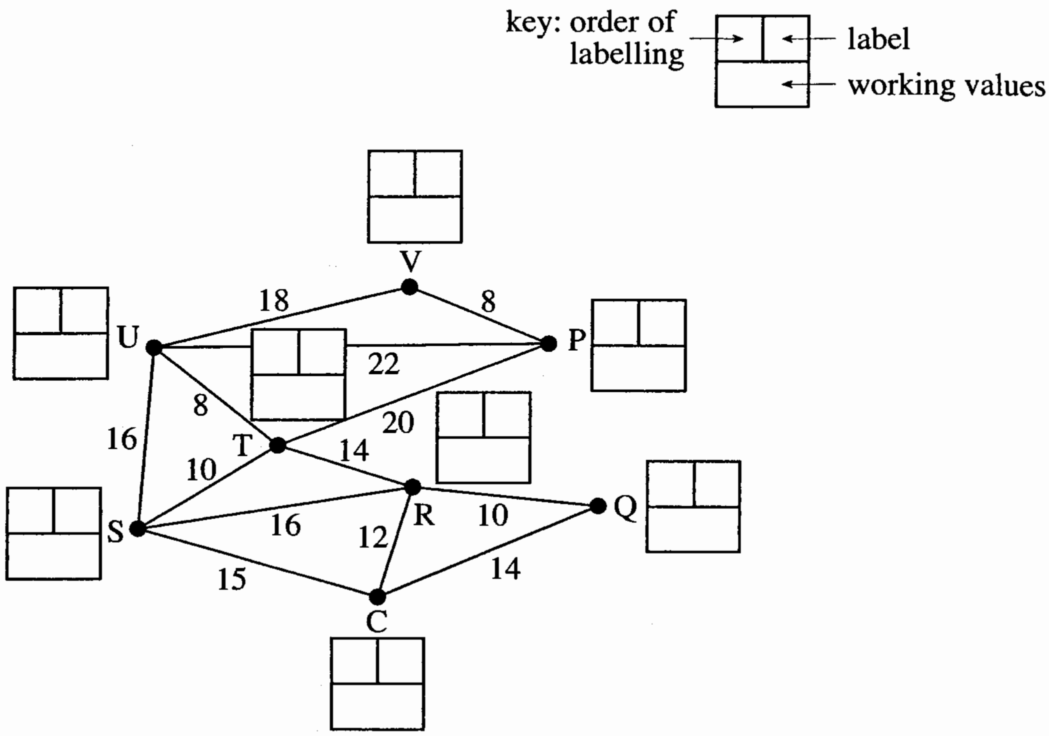


(ii)

(iii)

(iv)

4 **(i)**

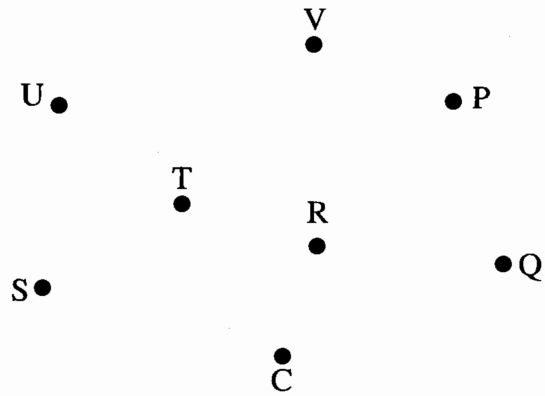


	PC	QC	RC	SC	TC	UC	VC
Shortest distances:							

Shortest route from P to C: ______________________________

Shortest route from V to C: ______________________________

(ii)



Order of including arcs: ______________________________

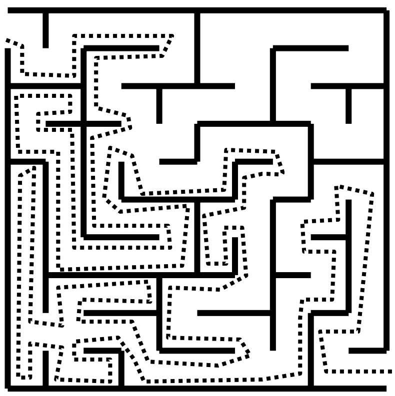
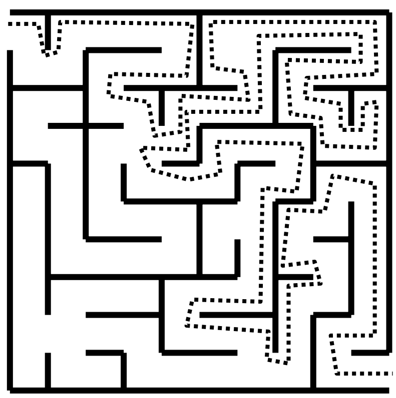
Length of connector: ______________________________

Mark Scheme 4771
June 2005

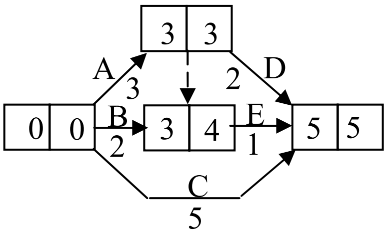
1.

(i) Any connected tree.	M1 A1
12 connections	B1
(ii) 14 connections	B1
(iii) e.g. He might be able to save cable by using it. e.g. To avoid overloading.	B1
(iv) Yes. A minimum connector is a tree. This gives the min number of arcs (n–1). This gives the minimum no of connections (2(n–1)).	B1 B1 B1

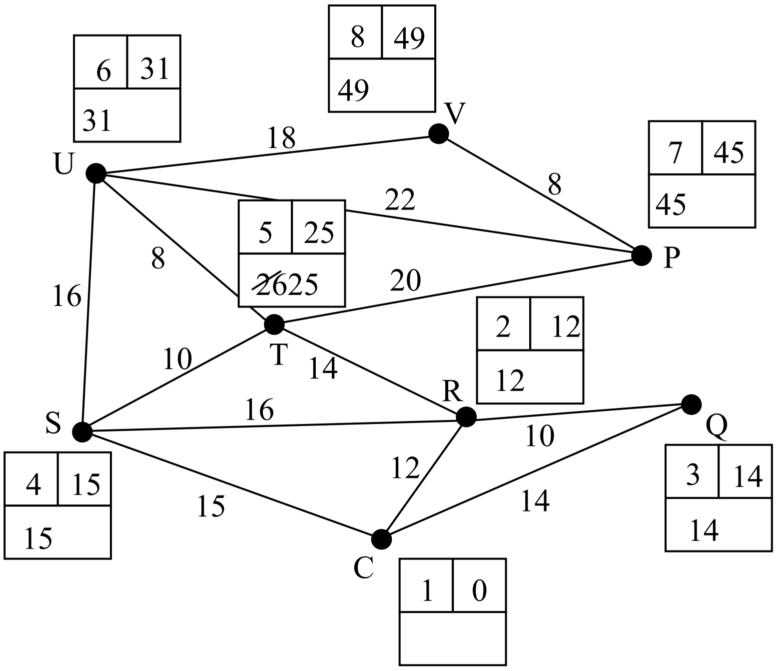
2.

(i) Janet John  	M1 A1 A1
(ii) Yes Janet's route traces west and south walls plus "attachments". John's route traces north and east walls plus "attachments". – or equivalent (Any "islands" are irrelevant.)	M1 A1 B1
(iii) Yes	B1
(iv) Yes All avenues covered by forward and backward pass (i.e. by John's original route + Janet's route).	B1

3.

(i) 	M1 A1 B1 B1
(ii) Critical − A, D and C	B1
(iii) Total float for B = 2 Independent float for B = 1 Total float for E = 1 Independent float for E = 0	B1 both total floats A1 B's independent A1 E's independent

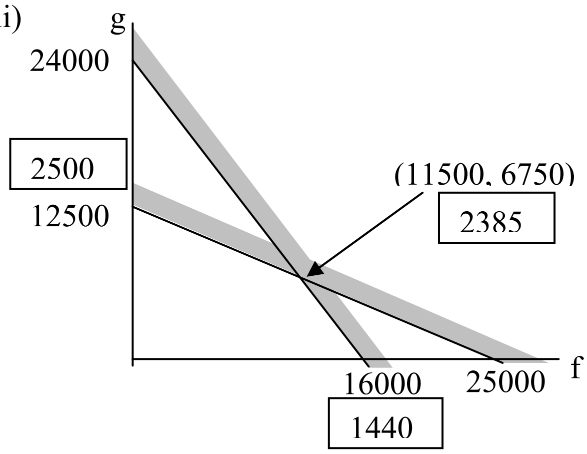
4.

(i)	B1 starting at C
	M1 Dijkstra A1 labels A1 order of labelling A1 working values
P Q R S T U V 45 14 12 15 25 31 49	B1
P T S C V U S C	B1 B1
(ii) PV ST CR RT UV Length = 80 TU QR	M1 A1 first 5 A1 last 2 B1 length
(iii) CP reduced to 26 CV reduced to 34	 B1 (both and no more)
(iv) UV replaced by PQ New length = 74	B1
(v) Q Semi-Eulerian. (Order of P changed from 3 to 4, but order of Q changed from 2 to 3 − so still 2 odd vertices.) or Cross the bridge and proceed as before or A valid route	M1 A1

5.

(i) eg. 00–19 → 0 20–49 → 1 50–69 → 2 70–84 → 3 85–99 → 4	M1 sca at proportions A1
(ii) 1, 0, 2, 3, 1, 3, 4, 3, 0, 0	M1 A1
(iii) eg. 00–15 → 0 16–39 → 1 40–63 → 2 64–95 → 3 96–99 → ignore	M1 missing some A1 times
(iv) 1, 0, 1, 0, 1, 1, 3, 3, 2, 2	B1 one ignored B1 rest
(v) Day 0 1 2 3 4 5 6 7 8 9 10 Stock 3 3 3 2 0 0 0 0 0 2 4 Disptd 0 0 0 0 1 0 2 1 0 0 0	M1 A1 A1
(vi) Day 0 1 2 3 4 5 6 7 8 9 10 Stock 3 3 3 2 0 1 0 0 1 3 5 Disptd 0 0 0 0 0 0 0 1 0 0 0	M1 using both ret dists A1 A1
Only 1 disappointed under new policy against 4 under old policy.	B1
Not definitely, but pretty convincingly.	B1

6.

(i) Let f be the number of litres of Flowerbase produced Let g be the number of litres of Growmuch produced	B1
Max $9f + 20g$ s.t. $0.75f + 0.5g \leq 12000$ $f + 2g \leq 25000$	M1 A1 M1 A1 A1
(ii) 	B1 labels + scales B1 B1 lines B1 shading
Max profit = £2500 by producing 12500 litres of Growmuch	M1 A1
(iii) No effect	B1
(iv) No effect The profit on Flowerbase will be reduced by more than that suffered by Growmuch, since it uses more fibre. The objective gradient will thus increase from −9/20, making it even less attractive to produce any Flowerbase.	M1 A1
(v) £3000	B1

4771 - Decision and Discrete Mathematics 1

General Comments

This paper was a slightly extended version of the paper set for 2620, and this report overlaps greatly with that of 2620.

Candidate performances were generally good – much better than has been the case in the past.

There was some evidence to suggest that some candidates spent far too long on question 5 and consequently ran out of time.

Comments on Individual Questions

1) **Graphs**

(i) Part (i) asked for the number of connections which the electrician has to make. However, many candidates gave the number of arcs in their network.

(ii) Those making the error referred to in part (i) usually added 1 to their answer, which was allowed.

(iii) Examiners do not expect candidates to show any detailed knowledge of the scenarios presented. Nothing is required beyond that which is given in the question. Thus they should not have been looking to their knowledge of domestic electricity circuits, nor bemoaning their lack of such knowledge, in attempting to answer part (iii). The issue here is that which has been considered in past examination papers – that introducing a new vertex into a network can have the effect of reducing the weight of the minimum connector.

(iv) Many candidates realised this was the case but found difficulty justifying it.

2) **Algorithms**

(i) Most candidates were successful with this question. Those that failed mostly allowed themselves to get stuck in a dead end.

(ii) That the algorithm does **not** leave one stuck in a dead end was not a sufficient answer to this question – that alone does not guarantee a route from entrance to exit. What was required was the recognition of the existence of two continuous connections between entrance and exit, the "northeast" wall (plus protuberances) and the "southwest" wall (plus protuberances).

(iii) Most said 'yes'.

(iv) Most said 'yes'. However too many answers concentrated on 'both sides of the walls' rather than routes. One was left with the impression that many had not realised that the maze was different from part (i).

3) **CPA**

(i) Most candidates were very successful with this question. Performance was much better overall than is usually the case on longer CPA questions set in context. A small number of candidates used activity on node (poorly) – the specification is clear that activity on arc is to be used.

(ii) Again done fairly well – most errors occurred when candidates had multiple end networks.

(iii) Generally disappointing. A significant number seemed to think that 'total' implied that floats, usually calculated incorrectly, had to be added together. Too many could not distinguish between 'total float' and 'independent float' conceptually, and/or failed to clarify what float they were actually evaluating.

4) **Networks**

(i) This was a very discriminating question. Good candidates started their Dijkstra from C. A significant minority started from P or V.

(ii) Kruskal is arguably the conceptually easiest algorithm on the syllabus. It might be expected that only the very weakest candidates would be unable to answer this question. However, rather more candidates then expected were not able to.

(iii) Very many candidates failed to score this mark by not providing an adequate answer. Noting that there will be a reduction in length is not an adequate answer to a question asking for the effect of a change. By how much, or to what, is required.

(iv) As per part (iii).

(v) Most candidates recognised the semi-Eulerian issue, if usually implicitly. Unsophisticated students gave a route as justification. Others noted the two odd nodes or pointed out that, since there was such a route from P to C before the bridge, then a route is now given by crossing the bridge and then following that original route.

5) **Simulation**

(i)(ii) Most candidates scored all 4 of these marks

(iii)(iv) A mixed response. Many recognised the need to discard some random numbers but choices of numbers discarded included various groups of numbers in the late 90s, several omitted 84-99 and a few 73-99. However too many used the whole range 00-99.

(v) This was answered quite well. Mistakes were easy to make, and were made, but most candidates showed a good understanding of what was needed.

(vi) Many candidates attempted to answer this question as per part (v), but with returns generated by the new distribution. In fact, the new distribution only comes into play after the number of laptops in stock drops to 2 or fewer. Thus the start of this simulation should be the same as the start of the simulation in part (v). It often was not.

6) **LP**

(i) A significant number of students had clearly run out of time when they started this question. Candidates exhibited all the usual weaknesses. At the worst extreme some identified variables (sometimes explicitly and sometimes implicitly) to do with fibre and nutrient, rather than with Flowerbase and Growmore. Less disastrously very many candidates failed adequately to define their variables (e.g. "Let x = Flowerbase and y = Growmore"), and many failed to note that the problem is a maximisation problem.

(ii) Too many candidates assumed that the optimal solution would be represented by the intersection of the two non-trivial constraint lines. It was disappointing to find a significant minority of candidates drawing graphs in their lined answer books – in several cases it appeared that centres did not make graph paper available to their students.

(iii) Not everyone who answered (B) correctly was able to provide an adequate justification.